Formulas: Inherent Availability and Reliability with Constant Failure and Repair Rates[1]

	Point (Instantaneous) Availability at time t	Average (Interval, Mission) Availability during the time period from t_1 to t_2	Limiting (Steady-State, Asymptotic) Availability as time becomes large
Inherent Availability[2]	$A(t)_{sys} = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}\right]^N$	$A(t_1,t_2)_{sys} = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 (t_2-t_1)} [e^{-(\lambda+\mu)t_1} - e^{-(\lambda+\mu)t_2}]\right]^N$ This is point availability averaged over the time interval.	$A_{inherent-sys} = \left[\frac{\mu}{\lambda+\mu}\right]^N = \left[\frac{MTBF}{MTBF+MTTR}\right]^N$
Reliability[3] (Availability = Reliability when μ = 0)		$R(t_2 \mid t_1)_{sys} = \left[e^{-(\lambda)(\Delta t)}\right]^N = e^{-N\lambda\Delta t}$ where $\Delta t = t_2 - t_1$ or $R(t)_{sys} = \left[e^{-\lambda t}\right]^N = e^{-N\lambda t}$ when $t_1 = 0$ and $t_2 = t$.	$R(t_2 \mid t_1) \to 0$ as $t_2 \to \infty$
Probability of r or less events[4]		$P = \sum_{n=0}^{r} \frac{e^{-N\lambda t}(N\lambda t)^n}{n!}$ where t is from 0 to t_2.	$P \to 0$ as $t_2 \to \infty$

Notes:

1 - Nomenclature: **N** = number of elements in a series configuration; **MTBF** = element mean time between failure; **MTTR** = element mean time to repair; **λ** = 1/MTBF; **μ** = 1/MTTR; and **t** = mission time. Note: λ and μ are constant rates over time--thus, the exponential distribution models the failure and repair distributions. λ, μ, and t have the same unit of time (e.g., hours). For design purposes, MTBF is a lower-bound parameter and MTTR is an upper-bound parameter.

2 - **A** = Availability, the probability that a repairable item is in an uptime state at a particular point in time or the average of point availability during a time interval from t_1 to t_2. Limiting (Steady-State) Availability typically occurs at the 6th decimal place when $(\lambda+\mu)*t$ is approximately 10 or more.

3 - **R** = Reliability, the probability of no failures during a time interval from t_1 to t_2. $R \le 0.3679$ when $N\lambda t \ge 1.0$. The notation $R(t_2 \mid t_1)$, a conditional probability, means the "reliability from t_1 to t_2 given the system has operated during the time interval from 0 to t_1 with no failures." The exponential distribution with a constant rate has no memory.

4 - **P** = the probability of r or less number of events (e.g., failures, repairs, etc.) during the time period from 0 to t. This probability is determined by the cumulative Poisson distribution. The Poisson process assumes failures are immediately repaired or replaced—thus, there is no accounting for repair time.

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